Wavelets and Fractals

April 26-28, 2010

Lectures and Abstracts

1. ANTOINE

   - Title: Unbounded Frames (Jean-Pierre Antoine, Peter Balazs, and Diana Stoeva)
   - Abstract: An unbounded frame is a generalized frame for which the lower frame bound vanishes or, equivalently, where the frame operator has an unbounded inverse. We study this structure, both in the continuous case and in the discrete case. In particular, we show that reconstruction is still possible. We also make the connection with frame multipliers.

2. ARNEODO (plenary)

   - Title: Master replication origins at the heart of the organization and fragility of the human genome
   - Abstract: During the course of evolution, mutations do not affect equally both strands of genomic DNA. This mainly results from asymmetric DNA mutation and repair processes associated with replication and transcription. In prokaryotes, prevalence of G over C and T over A is frequently observed in the leading replicating strand. The sign of the resulting TA and GC skews changes abruptly when crossing replication origin and termination sites, producing characteristic step-like transitions. In mammals, transcription-coupled skews have been detected but no bias had been associated with replication. In a first part, we present the analysis of intergenic and transcribed regions flanking experimentally identified human replication origins, demonstrating the existence of compositional strand asymmetries associated with replication. Wavelet-based multi-scale analysis of human genome skew profiles reveals numerous transitions allowing us to identify a set of one thousand putative replication initiation zones. Around these putative origins, the skew profiles display a remarkable pattern also observed in other mammalian genomes. Based on these results we propose a model of the mammalian replicon where termination sites are randomly distributed between adjacent origins. We report on DNA replication timing data that provide experimental verification of our in silico replication origin predictions. In a second part, we examine the organisation of the human genes around the replication origins. We show that replication origins, gene orientation and gene expression are not randomly distributed but on the opposite are at the heart of a strong organisation of human chromosomes. The analysis of open chromatin markers brings evidence of the existence of accessible open chromatin around the majority of the putative replication origins that replicate early in the S phase. We conclude by discussing the possibility that these "master" replication origins also play a key role in genome dynamics during evolution and in pathological situations like cancer.

3. BULTHEEL

   - Title: Geometric image approximation
   - Abstract: We shall consider the compression of piecewise smooth images. This means images that consist of (relatively few) smooth regions that have a smooth boundary. These boundaries are the only singularities in the image-function. It is well known that classical wavelets have a poor approximation behaviour on this kind of images. We present an approach that

has its origin in computer graphics. An image can be considered as a geometric object when the pixel values are z-values perpendicular to the xy-plane. A triangulation of the xy-plane gives a mesh whose vertices interpolate the object in 3D space. That mesh can be decorated for example by polynomial wavelets. Successive resolution levels are added as differences to that mesh when the triangulation is refined. In particular we shall measure the difference for each edge of the mesh along the normal bisector of the edge in a plane (though that edge) normal to the xy-plane. That is what is called a normal offset. The piercing point is where that bisector pierces the surface. When the piercing points are projected onto the xy-plane, then that will define what the next finer triangulation will be, etc. That triangulation will be anisotropic and will naturally adapt itself by stretching along the line(s) of singulartites in the image. This method is still in its infancy. We hall give a survey of this idea, some problems and some ideas about the proper coding which approaches the performance of JPEG2000. This presentation is base on the PhD thesis by Ward Van Aerschot cited below.

References

[1] W. Van Aerschot, M. Jansen, and A. Bultheel. Normal offsets for digital image compression. In F. Truchetet and O. Laligant, editors, *Optics East, Wavelet Applications in Industrial Processing III*, volume 6001 of *SPIE Proceedings*, pages 148–159. SPIE, October 2005. SPIE Optics East conference, Boston, MA, October 23-26, 2005.

[2] W. Van Aerschot, M. Jansen, and A. Bultheel. A nonlinear contour preserving transform for geometrical image compression. In J. McDonald, C. Markham, and J. Ghent, editors, *2007 International Machine Vision and Image Processing Conference (IMVIP 2007)*, pages 143–149. IEEE Computer Society Press, 2007.

[3] W. Van Aerschot, M. Jansen, and A. Bultheel. Piecewise smooth image compression using normal displacement maps. In *26th Picture Coding Symposium, Lisboa, Portugal*, November 2007.

[4] W. Van Aerschot, M. Jansen, and A. Bultheel. Normal mesh based geometrical image compression. *Image and Vision Computing*, 27(4):459–468, 2009. Available online July 15, 2008.

[5] W. Van Aerschot, M. Jansen, E. Vanraes, and A. Bultheel. Image compression using normal mesh techniques. In A. Cohen, J.-L. Merrien, and L.L. Schumaker, editors, *Curves and Surface Fitting, Avignon 2006*, pages 266–275, Brentwood, 2007. Nashboro Press.

[6] Ward Van Aerschot. *Multiscale Geometric Image Approximation (Multischaal benadering van geometrische beelden).* PhD thesis, K.U.Leuven, September 2009.

4. CLAUSEL

- Title: Some prevalent results about strongly monoHölder functions
- Abstract: A function f defined on some interval I is said to be strongly monoHölder with exponent $\alpha \in (0,1)$ if for some $C > 0$

$$\forall (x,y) \in I^2,\ |f(x) - f(y)| \leq C|x-y|^\alpha\,,$$

and

$$\forall J \subset I,\ osc(f,J) = \sup_{(x,y)\in J^2} (|f(y) - f(x)|) \geq \frac{1}{C}|J|^\alpha\,.$$

This notion can be generalized in a natural way to exponents greater than one.

A famous conjecture states that the Hausdorff dimension of such functions is equal to $2-\alpha$. Unfortunately, it is well–known that this conjecture is false in general. Therefore, we prove that this conjecture is true in a typical sense. Firstly, we show that "almost every" function

(in the sense of prevalence) of $C^\alpha(\mathbb{R}^d)$ is a strongly monoHölder function. Thereafter–always in the sense of prevalence–we prove that the Hausdorff dimension of the graph of "almost every" function of $C^\alpha(\mathbb{R}^d)$ equals $2-\alpha$.

5. DAUBECHIES (plenary)

   - Title: A very neat Analog-to-Digital converter
   - Abstract: The talk will start by reviewing the mathematical problem posed by practical analog-to-digital conversion, as well as mathematical results obtained in the last fifteen years about sigma-delta converters, which have intriguing links to sets (often with fractal boundaries) for which the integer translations tile R^n. It then goes to proposes a novel Nyquist-rate analog-to-digital conversion algorithm that achieves exponential accuracy in the bit-rate despite using imperfect components. The proposed algorithm is based on a robust implementation of a beta-encoder based on the golden ratio.

6. DEKKING (plenary)

   - Title: How fractal is the sum of two random fractals ?
   - Abstract: We consider two types of random Cantor sets, M-adic random Cantor sets, and Larsson's random Cantor sets. For both we will consider a probabilistic version of the Palis conjecture: does the sum of the Hausdorff dimensions lager than 1 imply that the algebraic difference of two independent copies contains an interval almost surely ?

7. DUVERNET

   - Title: Some mathematical results for testing the type of a semimartingale: Itō vs. multifractal
   - Abstract: Since the emergence of multifractal analysis, the issue of detecting multi-fractality in a signal has been a topic adressed in a vast number of research papers. While strong empirical arguments show that elaborate methods should be used in practice, there has been only few mathematical results concerning the theoretical performances of even simple estimators or test statistics for the hypothesis of multifractality. We present here the construction of a consistent test for H_0: "the signal belongs certain class of multifractal martingales" (namely the MRW's of Bacry and Muzy) against the alternative H_a: "the signal belongs to the class of Itō semimartingales", and conversely. This is a joint work with Mathieu Rosenbaum and Christian Y. Robert.

8. FLANDRIN (plenary)

   - Title: Empirical Mode Decomposition vs. wavelets for the analysis of scaling processes
   - Abstract: It has been recognized long ago that wavelet analysis is particularly well-adapted for studying scaling processes because of its built-in multiresolution structure. More recently, a different form of multiresolution decomposition, referred to as Empirical Mode Decomposition (EMD), has been proposed. In contrast with wavelets, EMD is not based on some fixed filtering scheme but is fully data-driven, with the ability of evidencing "natural" scales in a given process. It has also been shown that, in the case of broadband scaling processes such as fractional Brownian motion or fractional Gaussian noise, EMD acts spontaneously as a quasi-dyadic filterbank, thus allowing for new approaches to scaling assessment and estimation. The purpose of the talk will be to briefly review basics of EMD and to discuss in a comparative way wavelets and EMD when applied to the estimation of Hurst exponent and regularization fractal dimension.

9. HAMONIER

   - Title: Linear Multifractional Stable Motion : Wavelet Methods and Sample paths properties

- Abstract: The Linear Multifractional Stable Motion (LMSM), denoted by $Y = \{Y(t) : t \in \mathbb{R}\}$, is a Strictly α-Stable ($\mathcal{S}t\alpha\mathcal{S}$) stochastic process which was introduced in 2004 by Taqqu and Stoev [2] with a view to model some features of traffic traces on telecommunication networks, typically changes in operating regimes and burstiness (the presence of rare but extremely busy periods of activity). This process is obtained by replacing the constant Hurst parameter of the Linear Fractional Stable Motion by a function $H(\cdot)$. Throughout our talk, we will assume that the function $H(\cdot)$ takes values in $(1/\alpha, 1)$.

  Our goal is to improve some Taqqu and Stoev's results [3] concerning the sample path behavior of Y; to this end we will use a wavelet approach which is to a certain extent inspired from that in [1]. More precisely:

  (i) It has been shown in [3] that a sufficient condition for the trajectories of Y to be continuous, with probability 1, on a compact interval $\mathcal{K}$, is that $H(\cdot)$ be a Hölder function on $\mathcal{K}$. Also, it has been conjectured in the same article that this sufficient condition is not necessary, more precisely: the continuity of the trajectories of LMSM holds as long as $H(\cdot)$ is continuous. We will prove that this Taqqu and Stoev's conjecture is true.

  (ii) Under the same condition, some bounds of $\beta_Y(\mathcal{K})$, the critical uniform Hölder exponent of the trajectories of Y over $\mathcal{K}$, have been obtained in [3]; when $H(\cdot)$ belongs to the Hölder space $\mathcal{C}^\beta(\mathcal{K}, \mathbb{R})$ with $\beta > H^* := \max\{H(t), t \in \mathcal{K}\}$ then these bounds can be expressed as $H_* - 1/\alpha \leq \beta_Y(\mathcal{K}) \leq H_*$, where $H_* := \min\{H(t), t \in \mathcal{K}\}$. We will give a sharp modulus of continuity of the trajectories of Y and consequently prove that almost surely (a.s.) $\beta_Y(\mathcal{K}) = H_* - 1/\alpha$.

References

[1] Ayache, A., Roueff, F., Xiao, Y., *Linear fractional stable sheets: wavelet expansion and sample path properties*, Stochastic Process. Appl, 119, 1168-1197, 2009.

[2] Stoev, S., Taqqu, M.S., *Stochastic Properties of the Linear Multifractional Stable Motion*, Adv. Appl. Prob., 34, 1085-1115, 2004.

[3] Stoev, S., Taqqu, M.S., *Path Properties of the Linear Multifractional Stable Motion*, Fractals, 13, 2, 157-178, 2005.

10. JAFFARD

- Title: A grandcanonical multifractal formalism
- Abstract: cf. ROUX

11. LIAO

- Title: Some studies on dynamical diophatine approximation
- Abstract: The dynamical Diophantine approximation problems, which treat the approximation of the points by the orbits of a dynamical system, was introduced in a paper of Fan, Schmeling, and Troubetzkoy. In that paper, the authors fully studied the dynamical Diophantine approximation problems when the dynamical system is the doubling map on the unit interval. We generalize their results to expanding Markov maps of the interval with finite partitions. We also study some related questions for the irrational rotations. Some relations between the classical Diophantine approximation and the dynamical Diophantine approxiamation are explained.

12. MAMAN

- Title: Local behavior of traces of besov functions: prevalent results

- Abstract: Let $1 \leq d < D$ and (p, q, s) satisfying $0 < p < \infty$, $0 < q \leq \infty$, $0 < s - d/p < \infty$. I study the global and local regularity properties of traces, on affine subsets of $\mathbb{R}^D$, of functions belonging to the Besov space $B^s_{p,q}(\mathbb{R}^D)$. Given a d-dimensional subspace $\mathcal{H} \subset \mathbb{R}^D$, for almost all functions in $B^s_{p,q}(\mathbb{R}^D)$ (in the sense of prevalence), we are able to compute the singularity spectrum of the traces f_a of f on affine subspaces of the form $a+\mathcal{H}$, for Lebesgue-almostevery $a \in \mathbb{R}^{D-d}$. In particular, we prove that for Lebesgue-almost every $a \in \mathbb{R}^{D-d}$, these traces f_a are more regular than what could be expected from standard trace theorems, and that f_a enjoys a multifractal behavior.

13. ROUX

- Title: Toward a Multifractal Formalism for oscillating singularities (Stephane Jaffard, Stephane G. Roux and Patrice Abry)
- Abstract: Wavelet leaders have recently proved a key tool in the multifractal analysis of signals and images. However, their use was restricted to the determination of spectra of singularities associated with pointwise regularity exponents, such as the Hölder exponent. The notion of pointwise regularity alone is not enough to draw distinction between cusp-like and oscillating (chirp-like) singularities. We show that a definition of the oscillation exponent based on the notion of local fractional integral allow us to propose a variant of wavelet Leaders coefficients from which one obtains information on the oscillation exponents. We show that a multifractal formalism based on this new coefficients gives information on the density of the oscillation exponents present in a signal. After a theoretical part we present a numerical implementation of this new formalism for signals and images. The analysis of synthetic multifractal processes displaying or not oscillating singularities give promising results.

14. SCHMELING (plenary)

- Title: Joint orbits of rotations and phase transitions in diophantine approximation
- Abstract: We consider the transformation $(x, y) \to (x + \alpha, y + \beta)$ on the two-torus. For a fixed point $c \in [0, 1]$ and a number $\nu \in [1/2, \infty)$ we are interested in the set $\{y : \|n\alpha - x\| < \frac{1}{n^\nu}, \|n\beta - y\| < \frac{1}{\nu}$ for infinitely many $n\}$. We prove that the dimension of this set has a phase transition if ν crosses 1. This is joint work with Simon Kristensen.

15. SCHUCK JUNIOR

- Title: Analysis of Magnetic Resonance Spectroscopic Signals with Metabolite-based Autocorrelation Wavelets (Adalberto Schuck Jr., Aimamorn Suvichakorn, Christina Lemke, Jean-Pierre Antoine)
- Abstract: A new class of wavelet functions called metabolite-based autocorrelation wavelets is developed for analyzing Magnetic Resonance Spectroscopic (MRS) signals by means of the continuous wavelet transform (CWT), instead of the traditional wavelet like Morlet wavelet. These new wavelets are derived from the normalized autocorrelation function from metabolite data. The analytical part is developed based one a modified classical MRS FID signal model. Then, through numerical evaluation and using real metabolite data, they are created and used for detecting the presence of a given metabolite in a signal with a presence of many different components and for quantifying some of its parameters.

16. SLIMANE

- Title: Baire generic histograms of wavelet coefficients and large deviation formalism in Besov and Sobolev spaces
- Abstract: Histograms of wavelet coefficients are expressed in terms of the wavelet profile and the wavelet density. The large deviation multifractal formalism states that if a function f has a minimal uniform Hölder regularity then its Hölder spectrum is equal to the wavelet

density. The purpose is twofold. Firstly, we compute generically (in the sense of Baire's categories) these histograms in Besov $B_p^{s,q}(\mathbb{T})$ and $L^{p,s}(\mathbb{T})$ spaces, where $\mathbb{T}$ is the torus $\mathbb{R}^d/\mathbb{Z}^d$ (resp. in the Baire's vector space $V = \bigcap_{\varepsilon>0,p>0} B_p^{s(1/p)-\varepsilon/p,p}$ where $s : q \mapsto s(q)$ is a C^1 and concave function on R^+ satisfying $0 \leq s' \leq d$ and $s(0) > 0$). Secondly, as an application, we deduce some extra generic properties for the histograms in these spaces, and study the generic validity of the large deviation multifractal formalism in Besov and $L^{p,s}$ spaces for $s > d/p$ (resp. in the above space V). For details see [1].

References

[1] M. Ben Slimane, *The Genericity in the sense of Baire's category of the Histograms of Wavelet Coefficients*, J. Math. Anal. Appl. 349 (2009) 403–412.

17. STOEVA

- Title: Frames in Banach spaces
- Abstract: As it is well known, (Hilbert) frames play a crucial role in both pure and applied mathematics. From a practical point of view, it is interesting to know what happens when the frame elements are changed, under which conditions the new sequence is still a frame. Perturbation conditions which keep the Hilbert frame property have been already well investigated.

  In the present talk we consider some concepts which generalize frames to Banach spaces. Our attention is on Banach frames and X_d-frames (which generalize frames) and X_d-Bessel sequences (which generalize Bessel sequences). We present perturbation conditions which keep each of the mentioned concepts in Banach spaces. Our interest is on several conditions of "closeness" and the connection between them. We also discuss the necessity of these conditions.